MATHEMATICS SPECIALIST

MAWA Semester 1 (Unit 3) Examination 2017

Calculator-free

Marking Key

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The release date for this exam and marking scheme is

- **the end of week 8 of term 2, 2017**

Section One: Calculator-free (52 Marks)

Question 1(a)

Solution

$z_1 = i$, $z_2 = 2-3i$ and $z_3 = a-i$

$\Rightarrow$ (i) $z_1 - z_2 = i-(2-3i) = i-2+3i = -2+4i$

(ii) $z_3\overline{z_2} = (a-3i)\times(2+3i) = 2a+3ai+6i+9 = 2a+9+3(a+2)i$

(iii) $\dfrac{z_1 z_2}{z_3} = \dfrac{3+2i}{a-i}\times\dfrac{a-i}{a-i} = \dfrac{3a-3i+2ai+2}{a^2+1} = \dfrac{3a+2+(2a-3)i}{a^2+1} = \dfrac{3a+2}{a^2+1}+\dfrac{(2a-3)i}{a^2+1}$

Marking key/mathematical behaviours	Marks
(i)	
• Determines $z_1 - z_2 = -2+4i$	1
(ii)	
• Determines $\overline{z_2}$ and multiplies $z_3\overline{z_2}$	1
• Expresses the result in the form $a+bi$	1
(iii)	
• Indicates the need to multiply $\dfrac{z_1 z_2}{z_3}$ by $\dfrac{z_3}{z_3}$	1
• Multiplies this correctly	1
• Re-arranges in the form $a+bi$	1

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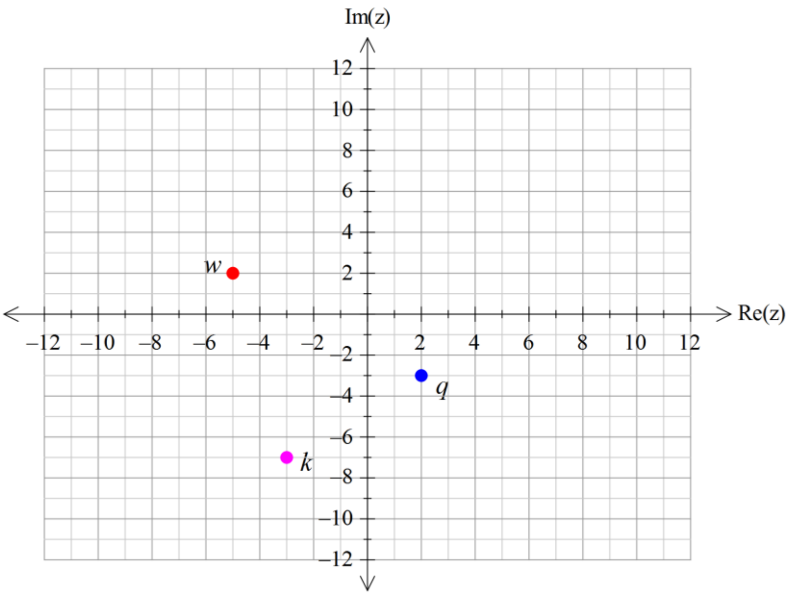
Question 1(a)

Solution



Marking key/mathematical behaviours	Marks
• Correctly plots q and w	1
• Calculates $k = -3 - 7i$	1
• Correctly plots k	1

Question 2(a)

Solution

$$z = 6 \times cis\left(\frac{\pi}{3}\right) = 6\left(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right) = 6\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = 3 + 3\sqrt{3}i$$

Marking key/mathematical behaviours	Marks
• substitutes the correct exact values into cis	1
• simplifies correctly	1

Question 2(b)

Solution

$$z^4 = \left(6 \times cis\left(\frac{\pi}{3}\right)\right)^4 = 6^4\left(cis\left(\frac{4\pi}{3}\right)\right) = 6^4\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 2^3 3^4(-1 - \sqrt{3}i) = 648(-1 - \sqrt{3}i)$$

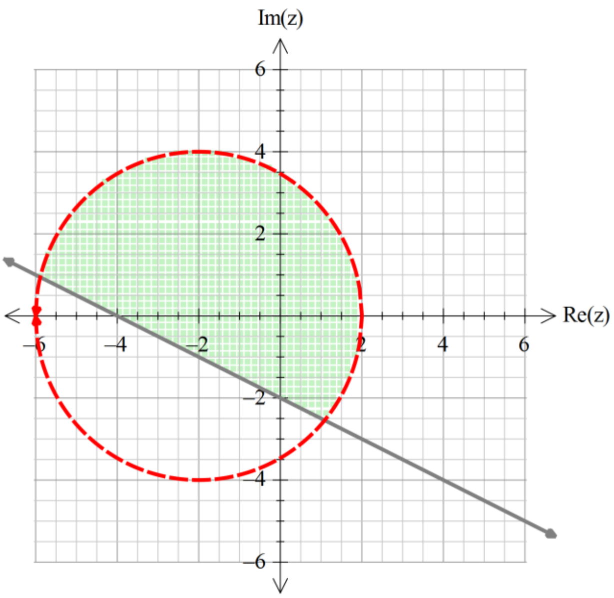
Marking key/mathematical behaviours	Marks
• applies de Moivre's theorem	1
• substitutes exact values	1
• simplifies	1

Question 2(c)

Solution

$\mathrm{Im}(z) \geq \frac{1}{2}\mathrm{Re}(z) - 2$ and

$(\mathrm{Re}(z) - 2)^2 + (\mathrm{Im}(z))^2 < 16$



Marking key/mathematical behaviours	Marks
• Correctly states inequation for half plane above the line	1
• Correctly states the inequality of the circular region	1
• Indicates that it is the intersection of the two regions (ie uses "and")	1
• Indicates the boundaries correctly by using the appropriate symbol within each inequation	1

Question 3(a)

Solution

$\sqrt{2x-2}$

$x \geq 1$

$y \geq 0$

Marking key/mathematical behaviours	Marks
• determines expression	1
• states domain	1
• states range	1

Question 3(b)

Solution

$$f \circ g(x) = \sqrt{2x-2}$$
$$x \geq 1$$
$$y \geq 0$$

Marking key/mathematical behaviours	Marks
• determines expression	1
• states domain	1
• states range	1

Question 3(c)

Solution

$$h(x) = 3(x-2)^2 - 1$$
$$y \geq -1$$

Marking key/mathematical behaviours	Marks
• rewrites function in turning point form	1
• states range	1

Question 3(d)

Solution

$$h(x) = 3(x-2)^2 - 1$$

Restricted domain: $x \leq 2$

$$y = 3(x-2)^2 - 1, \qquad x \leq 2,\ y \geq -1$$
$$x = 3(y-2)^2 - 1, \qquad y \leq 2,\ x \geq -1$$
$$(y-2)^2 = \frac{x+1}{3}$$
$$y = 2 - \sqrt{\frac{x+1}{3}}, \qquad y \leq 2$$
$$h^{-1}(x) = 2 - \sqrt{\frac{x+1}{3}}$$

Marking key/mathematical behaviours	Marks
• restricts domain correctly	1
• swaps x and y	1
• solves for $(y-2)^2$	1
• determines the correct inverse rule	1

Question 4

Solution

$x=-3,\ x=1$ are asymptotes: $f(x)=\dfrac{ax+b}{(x+3)(x-1)}$

$f(0)=2$: $\quad \dfrac{b}{-3}=2 \quad \Rightarrow \quad b=-6$

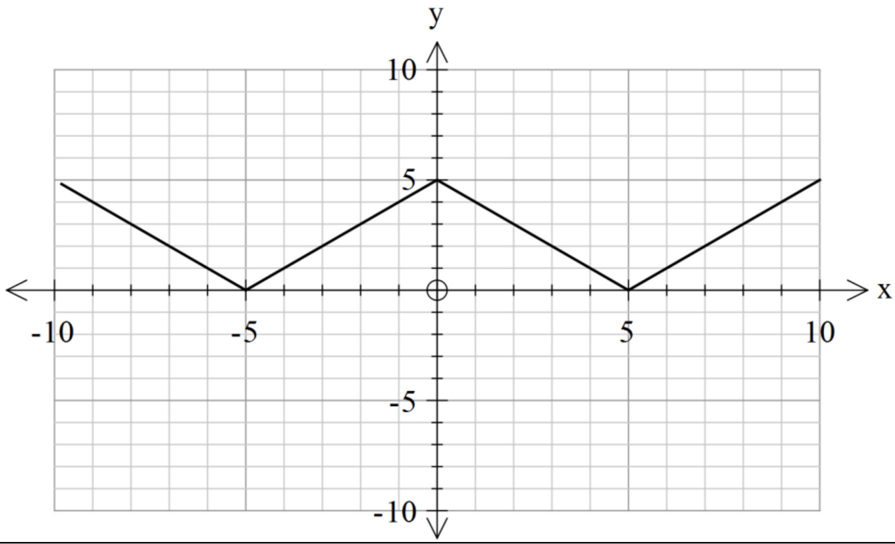
$f(-1)=2$: $\quad \dfrac{-a-6}{-4}=2 \quad \Rightarrow \quad a=2$

$$f(x)=\frac{2x-6}{(x-1)(x+3)}$$

Marking key/mathematical behaviours	Marks
• states values of c and d	1
• states value of b	1
• states value of a	1

Question 5(a)

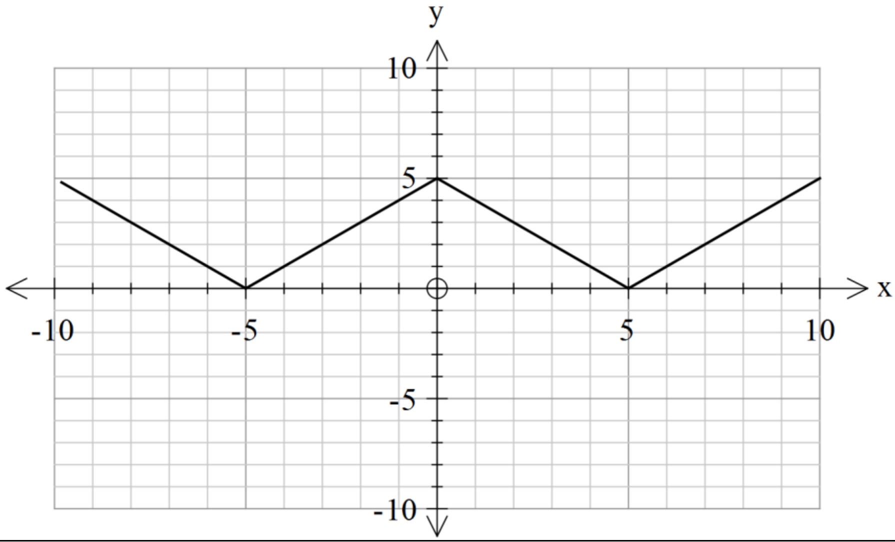
Solution



Marking key/mathematical behaviours	Marks
• sketches for $x>-5$	1
• sketches for $x\le -5$	1

Question 5(b)

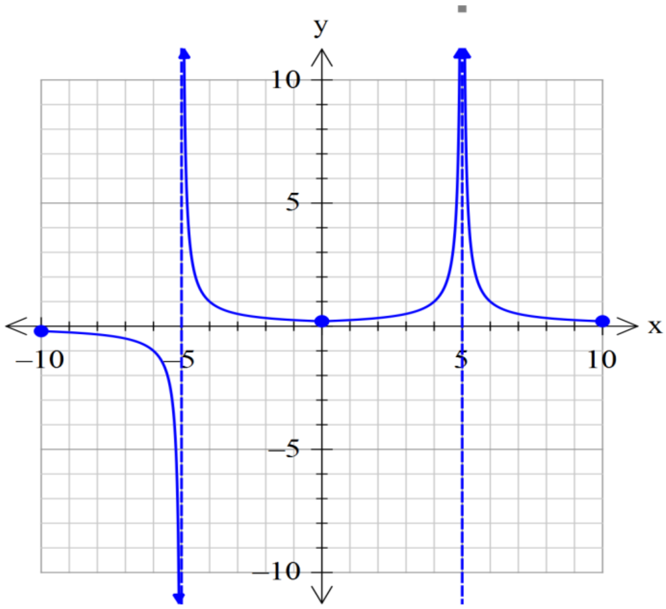
Solution



Marking key/mathematical behaviours	Marks
• sketches for $x > 0$	1
• sketches for $x \le 0$	1

Question 5(c)

Solution



Marking key/mathematical behaviours	Marks
• shows two asymptotes	1
• shows y intercept (=0.2)	1
• sketches correctly (shape and accuracy)	1+1

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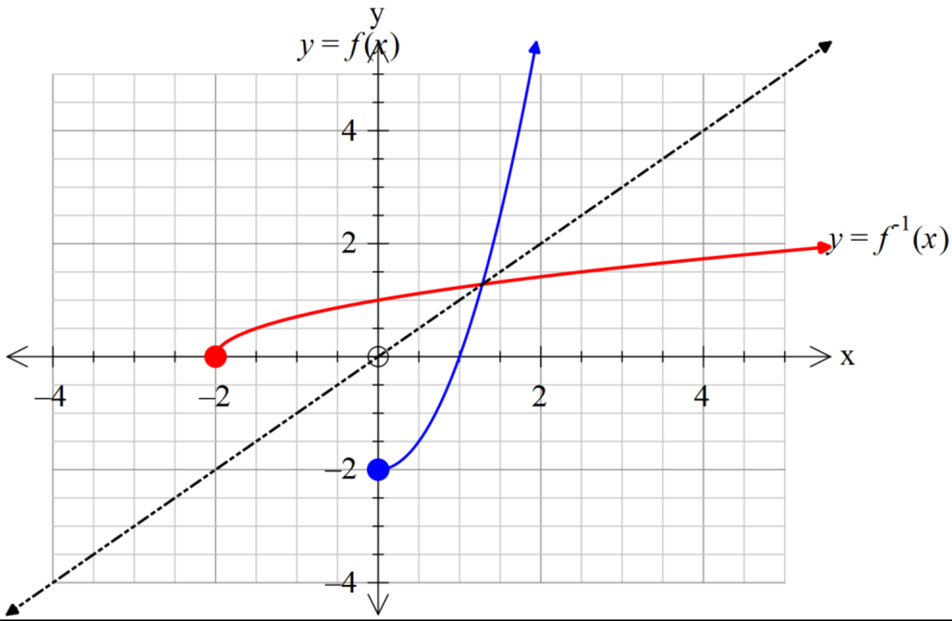
Question 6(a)

Solution



Marking key/mathematical behaviours	Marks
• sketch of inverse appears as a reflection in the line $y = x$	1
• shows correct end-point of (-2,0) and indicates continuation past $x = 5$	1
• sketch has a reasonably accurate shape (ie. crosses $f(x)$ at roughly the correct spot)	1

Question 6(b)

Solution

EITHER, uses point of intersection from graph, $x \approx 1.3$

OR, solves algebraically,

$$2x^2 - 2 = x$$

$$\Rightarrow \quad 2x^2 - x - 2 = 0$$

$$\Rightarrow \quad x = \frac{1+\sqrt{17}}{4}, \quad x \geq -2$$

Marking key/mathematical behaviours		Marks
EITHER	OR	
• uses point of intersection	establishes equation to solve	1
• states value near 1.3	states value	1

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Question 7(a)

Solution

$$\begin{aligned} x+3y-2z &= 3 \\ 4x+14y-3z &= 19 \\ 3x+12y+2z &= 21 \end{aligned} \Leftrightarrow \begin{aligned} x+3y-2z &= 3 \\ 2y+5z &= 7 \\ 3y+8z &= 12 \end{aligned} \Leftrightarrow \begin{aligned} x+3y-2z &= 3 \\ 2y+5z &= 7 \\ z &= 3 \end{aligned}$$

So $z = 3$, and back-substitution gives $y = -4$ and $x = 21$.

Marking key/mathematical behaviours	Marks
• systematically eliminates variables	1
• solves for z	1
• solves for y and x	1

Question 7(b)

Solution

$$\begin{aligned} x+3y-2z &= 3 \\ 4x+14y-3z &= 19 \\ 3x+12y+az &= b \end{aligned} \Leftrightarrow \begin{aligned} x+3y-2z &= 3 \\ 2y+5z &= 7 \\ 3y+(a+6)z &= b-9 \end{aligned}$$

$$\Leftrightarrow \begin{aligned} x+3y-2z &= 3 \\ 2y+5z &= 7 \\ (2a-3)z &= 2b-39 \end{aligned}$$

Infinitely many solutions when last equation reduces to $0z = 0$,
i.e. $a = 1.5$ and $b = 19.5$

Marking key/mathematical behaviours	Marks
• systematically eliminates variables	1
• uses the condition for infinitely many solutions	1
• solves for a and b	1

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